# Finite Math - Spring 2017

Lecture Notes - 2/10/2017

## Homework

- Section 3.2 - 57, 72, 73, 77

## Section 3.2 - Compound and Continuous Compound Interest

**Example 1.** *If* $\$1,000$ *is invested at* $6\%$ *interest compounded continuously, what is the value of the investment after* $8$ *years? Round answers to the nearest cent.*

**Solution.** *The principal is* $\$1,000$ *and the interest rate is* $r = 0.06$ *with a time of* $t = 8$ *years, so the future value in this case is*

$$A = \$1,000e^{(0.06)(8)} = \$1,000e^{0.48} = \$1,616.07$$

*which is we see is larger than any of the others.*

**Example 2.** *If* $\$2,000$ *is invested at* $7\%$ *compounded (a) daily, (b) continuously, what is the amount after* $5$ *years? How much interest is accrued in each case? Round answers to the nearest cent. (Assume* $365$ *days in a year.)*

**Solution.**

(a) $\$2838.04$ *with* $\$838.04$ *in interest.*

(b) $\$2838.14$ *with* $\$838.14$ *in interest.*

As before, we can use these compound interest models to figure out how much we should invest now to achieve a desired future value.

**Example 3.** *New parents are looking at a college savings account which gives* $8\%$ *interest. If they are looking to have* $\$80,000$ *when their child is ready to go to college in* $17$ *years, how much should they invest now if interest is compounded (a) semiannually, (b) continuously? Round answers to the nearest cent.*

**Solution.** *Here, we have* $r = 0.08$*,* $t = 17$*, and* $A = \$80,000$ *as given values. We are looking for the principal in both cases.*

(a) *If interest is compounded semiannually, then the interest per compounding period is* $i = \frac{0.08}{2} = 0.04$ *and the number of compounding periods is* $n = 2(17) = 34$*. So the formula gives us*

$$\$80,000 = P(1 + 0.04)^{34} = 3.794316P$$

*and solving for* $P$ *says that the principal the parents should invest is*

$$P = \$21,084.17.$$

(b) *If interest is compounded continuously, then*

$$\$80,000 = Pe^{(0.08)(17)} = 3.896193P$$

*so the principal is*

$$P = \$20,582.36.$$

**Example 4.** *You are looking at a retirement account which pays* 2% *interest. If you are looking to have* \$1,000,000 *in the account by the time you retire in* 50 *years how much should you invest now if interest is compounded (a) quarterly, (b) continuously? Round answers to the nearest cent.*

**Solution.**

(a) \$368,797.23

(b) \$367,879.44

We can also look to see how long something will take to mature given the principal, the growth rate, and the desired future value. The power rule for logarithms comes especially in handy here: $\log_b M^p = p \log_b M$.

**Example 5.** *How long will it take* \$10,000 *to grow to* \$25,000 *if it is invested at* 8% *compounded quarterly?*

**Solution.** *Here, we have* $P = \$10,000$, $A = \$25,000$, $r = 0.08$, $i = \frac{0.08}{4} = 0.02$, *thus the model gives us*

$$\$25,000 = \$10,000(1 + 0.02)^n$$

*and so we can solve for the number of compounding periods required.*

$$\begin{aligned} \$25,000 &= \$10,000(1 + 0.02)^n \\ 2.5 &= 1.02^n \\ \ln 2.5 &= n \ln 1.02 \\ n &= \frac{\ln 2.5}{\ln 1.02} = 46.27 \end{aligned}$$

*So, this means we need* 47 *quarters to achieve* \$25,000, *or* 11 *years and* 3 *quarters.*

**Example 6.** *How long will it take money to triple if it is invested at (a)* 5% *compounded daily? (b)* 6% *compounded continuously?*

**Solution.**

(a) 8,021 *days (about* 21.975 *years)*

(b) 18.310 *years*